Exercise 27

Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$x^2 - xy - y^2 = 1, \quad (2, 1), \quad \text{(hyperbola)}$$

Solution

The aim is to evaluate y' at $x = 2$ and $y = 1$ in order to find the slope there. Differentiate both sides of the given equation with respect to x.

$$\frac{d}{dx}(x^2 - xy - y^2) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(xy) - \frac{d}{dx}(y^2) = 0$$

$$(2x) - \left[\frac{d}{dx}(x)\right] y - x\left[\frac{d}{dx}(y)\right] - \left[2y \cdot \frac{d}{dx}(y)\right] = 0$$

$$2x - (1)y - x(y') - 2yy' = 0$$

Solve for y'.

$$2x - y = (x + 2y)y'$$

$$y' = \frac{2x - y}{x + 2y}$$

Evaluate y' at $x = 2$ and $y = 1$.

$$y'(2, 1) = \frac{2(2) - (1)}{(2) + 2(1)} = \frac{3}{4}$$

Therefore, the equation of the tangent line to the curve represented by $x^2 - xy - y^2 = 1$ at $(2, 1)$ is

$$y - 1 = \frac{3}{4}(x - 2).$$

Below is a graph of the curve and the tangent line at $(2, 1)$.